Condense into a single logarithm

1. \( \log 2x + \log 4x \)
   \[ \log 8x^2 \]

Solve

4. \( 3^{x+2} \cdot \left( \frac{1}{9} \right)^{-x+4} = 27^{2x-1} \)
   \[ \left( 3^2 \right)^{x+4} = (3^3)^{2x-1} \]
   \[ x+2 + 2x-8 = 6x-3 \]
   \[ 3x - 6 = 6x - 3 \]
   \[ -3 = 3x \]
   \[ -1 = x \]

6. \( \ln(x) = 5 \)
   \[ e^5 = x \]
   \[ 148.413 = x \]

8. \( \log_4 64^x = 1 \)
   \[ x \log_4 64 = 1 \]
   \[ x \left( \frac{3}{2} \right) = 1 \]
   \[ 3x = 1 \]
   \[ x = \frac{1}{3} \]

10. For a certain isotope, \( k = 0.0763 \). How long will it take 35 mg to become 210 mg? \( A = A_0 e^{kt} \)

   \[ 210 = 35 e^{-0.0763t} \]
   \[ 6 = e^{-0.0763t} \]
   \[ \ln 6 = -0.0763t \]
   \[ \frac{\ln 6}{-0.0763} = t \]
   \[ 23.483 = t \text{ years} \]
11. If a half-life for an isotope is 600 years, how long will it take for 800 mg to get down to 25 mg? $A = A_0e^{kt}$

Find $k$

\[
\frac{1}{2} = 1e^{k \cdot 600k} \\
\frac{1}{2} = e^{600k} \\
\ln \left(\frac{1}{2}\right) = 600k \\
\ln(25) - \ln(800) = 600k \\
\ln \left(\frac{25}{800}\right) = -0.0011...t \\
\ln \left(\frac{25}{800}\right) = -0.0011...t = t \\
3,000 \text{ years}
\]

12. $f(x) = \frac{1}{2}^{(x+4)} + 3$

a. What is the parent function?

\[y = \frac{1}{2}^x\]

b. Describe the Transformations.

\[\text{Left 4, Up 3}\]

c. Where is the Asymptote?

\[y = 3\]

d. Domain:

\[-\infty, \infty\]

e. Range:

\[(3, \infty)\]

13. $f(x) = -e^x + 4$

a. What is the parent function?

\[y = e^x\]

b. Describe the Transformations.

\[\text{Reflect over x, Up 4}\]

c. Where is the Asymptote?

\[y = 4\]

d. Domain:

\[-\infty, \infty\]

e. Range:

\[(-\infty, 4)\]
14. \( f(x) = \log_2(x - 4) - 4 \)
   a. What is the parent function?
      \[ y = \log_2 x \]
   b. Describe the Transformations.
      \[ \text{Right 4, Down 4} \]
   c. Where is the Asymptote?
      \[ x = 4 \]
   d. Domain:
      \[ (4, \infty) \]
   e. Range:
      \[ (-\infty, 0) \]

15. \( f(x) = -\ln(x + 2) + 4 \)
   a. What is the parent function?
      \[ y = \ln(x) \]
   b. Describe the Transformations.
      \[ \text{Reflect over } x, \text{ left 2, up 4} \]
   c. Where is the Asymptote?
      \[ x = -2 \]
   d. Domain:
      \[ (-2, \infty) \]
   e. Range:
      \[ (-\infty, 0) \]

Evaluate

16. \( \log_9 25 \approx \frac{\log 25}{\log 9} = 1.465 \)

17. \( \log_3 6 = \frac{\log 6}{\log 3} = 1.631 \)

18. Suppose \$2,500 is invested at 6% interest compounded quarterly. How long will it take for the amount to triple? \( A = p \left(1 + \frac{r}{n}\right)^{nt} \)

\[ 7500 = 2500 \left(1 + \frac{0.06}{4}\right)^{4t} \]

\[ 3 = \left(1.015\right)^{4t} \]

\[ \log_{1.015} 3 = 4t \]

\[ \left(\frac{\log 3}{\log 1.015}\right) / 4 = t \]

\[ 18.447 = t \text{ years} \]
19. Suppose $10 is invested at 8% interest compounded continuously. When will the investment be worth $100? \( A = Pe^{rt} \)

\[
\begin{align*}
100 &= 10e^{.08t} \\
10 &= e^{.08t} \\
\ln 10 &= .08t \\
\frac{\ln 10}{.08} &= t \\
\end{align*}
\]

\( t = 28.782 \) years

20. Of 30 milligrams of a radio active element, only 18 milligrams are left after 6.634 years. What is its half life? \( A = A_0e^{kt} \)

\[
\begin{align*}
18 &= 30e^{k(6.634)} \\
\frac{3}{5} &= e^k \\
\ln(\frac{3}{5}) &= 6.634k \\
\frac{\ln(\frac{3}{5})}{6.634} &= k \\
-.077 &= k \\
\end{align*}
\]

\( \frac{1}{2} = 1e^{-0.077t} \\
\ln(\frac{1}{2}) = -.077t \\
9.002 = t \) years

21. Of 500 grams of a certain isotope, 300 grams is left after 1.355 years. Find its half life. \( A = A_0e^{kt} \)

\[
\begin{align*}
300 &= 500e^{k(1.355)} \\
\frac{3}{5} &= e^k \\
\ln(\frac{3}{5}) &= 1.355k \\
\frac{\ln(\frac{3}{5})}{1.355} &= k \\
-.377 &= k \\
\end{align*}
\]

\( \frac{1}{2} = 1e^{-0.377t} \\
\ln(\frac{1}{2}) = -.377t \\
1.839 = t \) years

22. How long would it take to double an investment of $1,000 at 9\% interest compounded quarterly?

\[ A = p \left(1 + \frac{r}{n}\right)^{nt} \]

\[
\begin{align*}
2000 &= 1000 \left(1 + \frac{.09}{4}\right)^{4t} \\
2 &= 1.0225 \\
\log_{1.0225} 2 &= 4t \\
\frac{\log 2}{\log_{1.0225}} &= t \\
t &= 7.788 \text{ years}
\end{align*}
\]

23. Find the domain, intercept and vertical asymptote of \( f(x) = \ln(x+1) \)

- Domain: \(( -1, \infty )\)
- X-intercept: \((0,0)\)
- Vertical Asymptote: \(x = -1\)
Solve

24. \(10^{\log_{10}(3x-2)} = 25\)
\[3x-2 = 25\]
\[3x = 27\]
\[x = 9\]

26. \(\log_4(x + 3) + \log_4(x - 3) = \log_4 16\)
\[\log_4 (x^2 - 9) = \log_4 16\]
\[x^2 - 9 = 16\]
\[x^2 = 25\]
\[x = -5, 5\]

28. \(4^{4y-6} = 3^{2y+5}\)
\[(4y-6)\log_4 3 = (2y+5)\log_3 3\]
\[5y\log_4 3 - 6\log_4 3 = 2y\log_3 3 + 5\log_3 3\]
\[5y\log_4 3 - 2y\log_3 3 = 5\log_3 3 + 6\log_4 3\]
\[y = \frac{5\log_3 3 + 6\log_3 3}{5\log_4 3 - 2\log_3 3}\]
\[y = 2.917\]

30. \(\ln e^{3x} = 18\)
\[3x = 18\]
\[x = 6\]

31. \(\log_7 2401 = x\)
\[\frac{\log_{10} 2401}{\log_{10} 7} = x\]
\[4 = x\]

32. \(\log_x 5 = \frac{1}{4}\)
\[x^{1/4} = 5\]
\[x = 625\]

33. \(\log p = \frac{1}{2} \log 81\)
\[\log p = \log 9\]
\[p = 9\]
34. \(9^{\log_{3}(3x+1)} = 31\)

\[
\begin{align*}
3x + 1 &= 31 \\
3x &= 30 \\
x &= 10
\end{align*}
\]

35. \(\log_{3} 10 = \log_{3} 2x\)

\[
\begin{align*}
10 &= 2x \\
5 &= x
\end{align*}
\]

36. \(\log(x^2 + 9x) = \log 10\)

\[
\begin{align*}
0 &= x^2 + 9x - 10 \\
0 &= (x + 10)(x - 1) \\
x &= -10, 1
\end{align*}
\]

37. \(\log_{3} 7 + \log_{3} x = \log_{3} 14\)

\[
\begin{align*}
7x &= 14 \\
x &= 2
\end{align*}
\]

38. \(\log_{4}(x + 2) + \log_{4}(x - 4) = 2\)

\[
\begin{align*}
\log_{4}(x^2 - 2x - 8) &= 2 \\
4^2 &= x^2 - 2x - 8 \\
16 &= x^2 - 2x - 8 \\
0 &= x^2 - 2x - 24 \\
0 &= (x - 6)(x + 4) \\
x &= 6, -4
\end{align*}
\]

39. \(3^x = 55\)

\[
\begin{align*}
\log_{3} 55 &= x \\
\frac{\log 55}{\log 3} &= x \\
3.648 &= x
\end{align*}
\]

40. \(2.1^{x-5} = 9.32\)

\[
\begin{align*}
\log_{2.1} 9.32 &= x - 5 \\
\log_{2.1} 9.32 + 5 &= x \\
8.689 &= x
\end{align*}
\]

41. \(\frac{1}{27} = 3^{2x + 5}\)

\[
\begin{align*}
3^{-3} &= 3^{2x + 5} \\
-3 &= 2x + 5 \\
-8 &= 2x \\
x &= -4
\end{align*}
\]

42. \(\log_{2}(5x - 3) - \log_{2}(x^2 - 3) = 1\)

\[
\begin{align*}
\log_{2}\left(\frac{5x - 3}{x^2 - 3}\right) &= 1 \\
2^1 &= \frac{5x - 3}{x^2 - 3} \\
2x^2 - 6x &= 5x - 3 \\
2x^2 - 5x - 3 &= 0 \\
2x^2 - 6x + x - 3 &= 0 \\
2x(x - 3) + 1(x - 3) &= 0 \\
(x - 3)(2x + 1) &= 0 \\
x &= 3, -\frac{1}{2}
\end{align*}
\]