Oblique Triangle: a triangle with no rt. <

Law of Sines:
\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

This law works when you have:
- AAS
- ASA
- SSA

**AAS**

1. \[
\frac{a}{\sin 20^\circ} = \frac{12}{\sin 110^\circ}
\]
\[
a \cdot \sin 110^\circ = 12 \cdot \sin 20^\circ
\]

**ASA**

2. Given \( A = 100^\circ, \ c = 24, \ B = 20^\circ \)

\[
\frac{a}{\sin 100^\circ} = \frac{24}{\sin 60^\circ}
\]
\[
b = \frac{24}{\sin 20^\circ} = \frac{24}{\sin 60^\circ}
\]

\[
\angle C = 60^\circ
\]
\[
\frac{a}{\sin 20^\circ} = \frac{c}{\sin 60^\circ}
\]
8.1 Law of Sines

SSA

3. Given $A = 70^\circ$, $a = 9$, $b = 14$

$$\frac{14}{\sin B} = \frac{9}{\sin 70^\circ}$$

$$9 \sin B = 14 \sin 70^\circ$$

$$\sin B = \frac{14 \sin 70^\circ}{9}$$

$$B = \sin^{-1}(1.462)$$

$$B = \sin^{-1}(1.506)$$

SSA and the Ambiguous Case

Whenever given SSA you must test to make sure it works. There are three possible outcomes for a SSA triangle:

1. No triangle
2. 1 Triangle (solution)
3. 2 Triangles (solution)

To test you need to find the altitude (or height) of the triangle. The side in question needs to be greater than or equal to the altitude.

No Triangle:

Given $A = 70^\circ$, $a = 12$, $b = 20$

$$\frac{20}{\sin B} = \frac{12}{\sin 70^\circ}$$

$$12 \sin B = 20 \sin 70^\circ$$

$$\sin B = \frac{12}{10}$$

$$B = \sin^{-1}(1.506)$$
One Triangle:
Given $A = 30^\circ$, $a = 12$, $b = 24$
\[
\frac{24}{\sin B} = \frac{12}{\sin 30^\circ}
\]
\[
C = 20.785
\]

Given $A = 25^\circ$, $a = 18$, $b = 16$
\[
\frac{16}{\sin B} = \frac{18}{\sin A}
\]
\[
C = 31.182
\]

Two Triangles:
Given $A = 42^\circ$, $a = 18$, $b = 24$
\[
\frac{24}{\sin B} = \frac{18}{\sin 42^\circ}
\]
\[
B = 63.148^\circ
\]
\[
C = 180 - 63.148^\circ
\]
Solve each of the following:

1. Given $A = 62^\circ, a = 10, b = 12$
   
   \[
   \frac{10}{\sin 62^\circ} = \frac{12}{\sin B}
   \]
   
   \[10 \sin B = 12 \sin 62^\circ \]
   
   $B = \text{error}$

2. Given $A = 42^\circ, B = 58^\circ, b = 14$
   
   \[
   \frac{14}{\sin 58^\circ} = \frac{a}{\sin 42^\circ}
   \]
   
   \[a \sin 58^\circ = 14 \sin 42^\circ \]

3. Given $A = 98^\circ, a = 10, b = 3$
   
   \[
   \frac{10}{\sin 98^\circ} = \frac{3}{\sin B}
   \]

4. Given $A = 31^\circ, a = 29, b = 46$
   
   \[
   \frac{29}{\sin 31^\circ} = \frac{46}{\sin B}
   \]
5. Given $A = 47^\circ, B = 95^\circ, c = 5$

\[ \angle C = 38^\circ \]
\[ a = 5.940 \]
\[ b = 8.090 \]

**Area of an Oblique Triangle:**

\[ A = \frac{1}{2} ab \sin C \]

Find the area given $a = 80$, $b = 41$, $C = 100^\circ$

\[ A = \frac{1}{2} (41)(80) \sin 100 \]
\[ A = 1615.085 \]

6. Find area given $B = 120^\circ$, $a = 32$, $c = 50$

\[ A = \frac{1}{2} (50)(32) \sin 120 \]
\[ A = 692.820 \]
Applications

7. The bearing from the Pine Fire Tower to the Colt Fire Tower is N 65° E, and the two towers are 30 km apart. A fire spotted by the rangers in each tower has a bearing of N 80° E from Pine and S 70° E from Colt. Find the distance of the fire from each tower.

\[
\frac{30}{\sin 30} = \frac{x}{\sin 85}\]

\[x = 42.426 \text{ km}\]

\[\frac{30}{\sin 30} = \frac{y}{\sin 15}\]

\[y = 15.529 \text{ km}\]