6.6-6.7, 8.1-8.2 Test REVIEW

6.6-6.7:

Solve each of the following triangles given that C is a right angle.

1. $a = 24, b = 15$
   \[ c^2 = a^2 + b^2 \]
   \[ c = 30 \]
   \[ \tan A = \frac{24}{15}, \tan B = \frac{15}{24} \]
   \[ A = \tan^{-1} \left( \frac{24}{15} \right), B = \tan^{-1} \left( \frac{15}{24} \right) \]
   \[ C = 840, A = 57.99, B = 32.01^\circ \]

2. $a = 11, b = 3$
   \[ c^2 = a^2 + b^2 \]
   \[ c = 11.5 \]
   \[ \tan A = \frac{3}{11}, \tan B = \frac{11}{3} \]
   \[ A = \tan^{-1} \left( \frac{3}{11} \right), B = \tan^{-1} \left( \frac{11}{3} \right) \]
   \[ C = 180, A = 74.14, B = 15.86^\circ \]

3. $A = 36^\circ, b = 10$
   \[ c = \frac{10}{\cos 36^\circ} \]
   \[ C = 74.01, A = 36^\circ, B = 54^\circ, c = 52.26 \]

4. $a = 12, B = 50^\circ$
   \[ \cos 50 = \frac{12}{c} \]
   \[ c = 19.5 \]

5. $c = 19, b = 12$
   \[ a^2 = 12^2 + 19^2 \]
   \[ a = 22.5 \]
   \[ \tan B = \frac{12}{19}, \tan C = \frac{19}{12} \]
   \[ B = \tan^{-1} \left( \frac{12}{19} \right), C = \tan^{-1} \left( \frac{19}{12} \right) \]
   \[ a = 21.7, \angle A = 50.83^\circ, \angle B = 39.16^\circ \]

6. $\tan A = 9, b = 2$
   \[ \tan A = \frac{9}{2}, \tan B = \frac{2}{9} \]
   \[ a = 18, \angle A = 83.66^\circ, \angle B = 6.34^\circ \]

7. $\sin B = \frac{3}{4}, b = 6$
   \[ \sin B = \frac{3}{4} \]
   \[ B = \sin^{-1} \left( \frac{3}{4} \right), A = \cos^{-1} \left( \frac{3}{4} \right) \]
   \[ a = 2.57, \angle A = 41.41^\circ, \angle B = 48.59^\circ \]

8. $\tan B = 2, a = 4$
   \[ \tan B = 2 \]
   \[ a = 4, \angle A = \tan^{-1} \left( \frac{1}{2} \right), \angle B = \tan^{-1} \left( \frac{1}{2} \right) \]
   \[ b = 8, \angle A = 26.56^\circ, \angle B = 63.43^\circ \]

9. A ladder is placed 50 feet from the base of the wall. If the ladder makes an angle of 40° with the ground, what is the length of the ladder?

\[ \cos 40 = \frac{50}{x} \]
\[ x = \frac{50}{\cos 40^\circ} \]
\[ x = 65.27 \text{ feet} \]
10. From the top of a 60ft lighthouse a man looks down at a boat at an angle of depression of 30°. How far away is the boat from the lighthouse?

\[ \tan 60° = \frac{y}{60} \]

\[ x = 103.923 \text{ ft} \]

11. A zip line is 100 meters long, and travels a horizontal distance of 70 feet. What is the angle of elevation from the end of the zip line to where it starts?

\[ \cos x = \frac{70}{100} \]

\[ x = 45.573° \]

Evaluate the following: in radians

12. \( \arcsin(0.8) \)

\[ = 0.927 \]

13. \( \sin(\arcsin(0.8)) \)

\[ = 0.8 \]

14. \( \arcsin(\sin(\pi)) \)

\[ = 0 \]

14. If \( \arcsin(x) \) is undefined, what do we know about \( x \)?

\[ x \text{ is not in the domain } [-1, 1] \]

Find the Domain and Range for each

15. \( y = \sin x \)

\[ D: (-\infty, \infty) \]

\[ R: [-1, 1] \]

16. \( y = \cos x \)

\[ D: (-\infty, \infty) \]

\[ R: [-1, 1] \]

17. \( y = \tan x \)

\[ D: (-\frac{\pi}{2}, \frac{\pi}{2}) \]

\[ R: (-\infty, \infty) \]

18. \( y = \arcsin x \)

\[ D: [-1, 1] \]

\[ R: [-\frac{\pi}{2}, \frac{\pi}{2}] \]

19. \( y = \arccos x \)

\[ D: [-1, 1] \]

\[ R: [0, \pi] \]

20. \( y = \arctan x \)

\[ D: (-\infty, \infty) \]

\[ R: \left( -\frac{\pi}{2}, \frac{\pi}{2} \right) \]

Using triangles evaluate each of the following. Your answer must be a fraction.

21. \( \sin \left( \arccos \frac{4}{5} \right) \)

\[ \frac{3}{5} \]

22. \( \sec \left( \arcsin \frac{5}{7} \right) \)

\[ \frac{7}{2\sqrt{6}} = \frac{7\sqrt{6}}{12} \]
8.1-8.2:

Solve the following triangles, given they are oblique.

23. \( A = 22^\circ, B = 121^\circ, a = 17 \) 

\[ \begin{align*} \frac{17}{\sin 22^\circ} &= \frac{b}{\sin 121^\circ} \\ b &= \frac{17 \sin 121^\circ}{\sin 22^\circ} \\ &= \frac{17 \sin 121^\circ}{\sin 22^\circ} \approx 38.899 \\ \frac{17}{\sin 22^\circ} &= \frac{c}{\sin 37^\circ} \\ c &= \frac{17 \sin 37^\circ}{\sin 22^\circ} \\ &= \frac{17 \sin 37^\circ}{\sin 22^\circ} \\ \angle A &= 22^\circ, \angle B &= 121^\circ, \angle C = 37^\circ \\
\end{align*} \]

24. \( B = 150^\circ, a = 10, b = 3 \) 

\[ \frac{3}{\sin 150^\circ} = \frac{10}{\sin A} \]

\( h = 5 \)

No Solution

25. \( A = 11^\circ, b = 22, c = 21 \) 

\[ \begin{align*} a^2 &= 22^2 + 21^2 - 2(22)(21)\cos 11^\circ \\ &= 484 + 441 - 924 \cos 11^\circ \\ &= 925 - 924 \cos 11^\circ \\ &= 4.246 \\
\frac{4.246}{\sin 11^\circ} &= \frac{21}{\sin C} \\
C &= 70.912^\circ \\
a &= \sqrt{484 + 441 - 924 \cos 11^\circ} \\
&= \sqrt{925 - 924 \cos 11^\circ} \\
&= 22.40 \\
B &= 98.088^\circ, \angle C = 70.912^\circ \\
\end{align*} \]

26. \( B = 150^\circ, a = 10, b = 3 \)

No Solution

27. \( B = 25^\circ, a = 6.2, b = 4 \)

\[ \begin{align*} \frac{4}{\sin 25^\circ} &= \frac{6.2}{\sin A} \\
A &= \sin^{-1} \left( \frac{6.2}{4} \sin 25^\circ \right) \\
&= 40.924^\circ, \angle B = 139.076^\circ \\
\angle C &= 114.067^\circ, \angle C = 8.64^\circ \\
c &= 2.60 \\
\end{align*} \]

Option 1: \( \angle A = 40.924^\circ, \angle B = 139.076^\circ, \angle C = 8.64^\circ \)

Option 2: \( \angle A = 114.067^\circ, \angle C = 2.60 \)

28. \( a = 6, b = 8, c = 9 \)

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\[ \begin{align*} \angle A &= \sin^{-1} \left( \frac{6}{9} \right) \\
&= 40.804^\circ, \angle B = 60.61^\circ, \angle C = 78.582^\circ \\
\end{align*} \]

29. \( A = 27^\circ, b = 5, c = 7 \)

\[ \begin{align*} A &= \frac{1}{2} (5)(7) \sin 27^\circ \\
A &= 7.945 \\
\end{align*} \]

30. \( B = 80^\circ, a = 4, c = 8 \)

\[ \begin{align*} A &= \frac{1}{2} (4)(8) \sin 80^\circ \\
A &= 15.757 \\
\end{align*} \]
31. To approximate the length of a pond, a surveyor marks two points (A & C) at either end of the pond. She walks 425 meters from point A to point B (on the side of the pond). She then turns 65° and walks 300 meters to point C. Find the length of the pond.

\[ x^2 = 425^2 + 300^2 - 2(425)(300)(\cos 115°) \]

\[ x = 615.136 \text{ m} \]

32. A tree stands on a hillside of a slope of 28°. From a point 75 feet down the hill the angle of elevation to the top of the tree is 45°. What is the height of the tree?

\[ \cos 28° = \frac{a}{75} \]

\[ a = 66.221 \]

\[ \sin 28° = \frac{b}{75} \]

\[ b = 35.210 \]

\[ \tan 45° = \frac{c}{66.221} \]

\[ c = 66.221 \]

\[ h = 31.011 \text{ ft} \]

33. Two ships leave a port at the same time. One travels at a bearing of N45°W at 4 mph. The others at a bearing of S34°W at 10 mph. How far apart are the ships after 3 hours?

\[ x^2 = 12^2 + 30^2 - 2(12)(30)(\cos 16°) \]

\[ x = 34.371 \text{ miles} \]