5.1 Exponential Functions

Date: __________

Identify the pattern of each:

1. $4, -1, -6, ...$
   \[ y = 5 \cdot (-5)^x \]

2. $4, 8, 32, ...$
   \[ y = 2^x \]

3. $2, 6, 18, ...$
   \[ y = 3^x \]

4. $27, 3, \frac{1}{3}, ...$
   \[ y = \frac{1}{9} \cdot \left(\frac{1}{3}\right)^x = \left(\frac{1}{3}\right)^{x+2} \]

General form of an Exponential Function:

Exponential Growth

\[ y = a \cdot b^x \]

Exponential Decay

\[ y = a \cdot \frac{1}{b^x} \]

\[ y = b^x \]

Where $b > 1$

\[ y = b^x \]

Where $0 < b < 1$

Both graphs have an asymptote $y = 0$. WHY?

There is no exponent that will result in a 0 or negative.

Domain: $(-\infty, \infty)$
Range: $(0, \infty)$

What is $e$?

\[ e \approx 2.718 \ldots \]
Exponential Functions in Finance: Loans, savings accounts, depreciation all work with exponential functions.

Compound interest: interest calculated on the initial principal and also on the accumulated interest of previous periods of a deposit or loan.

Formulas:  
Set # of compounds: \( A = P \left(1 + \frac{r}{n}\right)^{nt} \)  
Compounds continuously: \( A = Pe^{rt} \)

Where \( A \): account balance, \( P \): Principal, \( r \): rate, \( n \): number of times compounded, \( t \): time

Example: A total of $14,000 is invested in a savings account with an annual interest of 8%. Which of the compound options below would yield the best profit, given the money will be in the account for 12 years.

Option 1: Compound 1 time a year
\[
A = \frac{14000 \times (1 + 0.08)}{1} \\
A = 14000(1 + 0.08) \\
A = 14000(1.08) \\
A = \$14,960
\]

Option 2: Compound 2 times a year
\[
A = \frac{14000 \times (1 + 0.08/2)}{2} \\
A = 14000(1 + 0.04) \\
A = 14000(1.04) \\
A = \$14,560
\]

Option 3: Compound 4 times a year
\[
A = \frac{14000 \times (1 + 0.08/4)}{4} \\
A = 14000(1 + 0.02) \\
A = 14000(1.02) \\
A = \$14,580
\]

Option 4: Compound 12 times a year
\[
A = \frac{14000 \times (1 + 0.08/12)}{12} \\
A = 14000(1 + 0.006667) \\
A = 14000(1.006667) \\
A = \$14,592
\]

Option 5: Compounded continuously
\[
A = Pe^{rt} \\
A = 14000e^{0.08 \times 12} \\
A = \$15,632
\]

Best option: 5
Graphing Exponential Functions with Transformations.

What movement will affect the asymptote?

1. $f(x) = (2^{x-1}) - 3$
   - Parent function: $y = 2^x$
   - Asymptote: $y = -3$
   - Domain: $(-\infty, \infty)$
   - Range: $(-3, \infty)$

2. $f(x) = -\left(\frac{1}{2}^{x+2}\right) + 4$
   - Parent function: $y = \frac{1}{2}^x$
   - Asymptote: $y = 4$
   - Domain: $(-\infty, \infty)$
   - Range: $(-\infty, 4)$

3. $f(x) = 2\left(\frac{1}{2}^{4-x}\right)$
   - Parent function: $y = 4^x$
   - Asymptote: $y = 2$
   - Domain: $(-\infty, \infty)$
   - Range: $(2, \infty)$