1.4 Solving Quadratic Equations

Date: ____________

Zero Product Property:

\[ \text{if } ab = 0 \text{ then } a = 0 \text{ or } b = 0 \]

Solve by factoring!

1) \( k^2 - 8k = 0 \)
   \[ k(k-8) = 0 \]
   \[ k = 0 \text{ or } k-8 = 0 \]
   \[ k = 0 \text{ or } k = 8 \]

2) \( n^2 - 8n + 15 = 0 \)
   \[ (n-5)(n-3) = 0 \]
   \[ n-5 = 0 \text{ or } n-3 = 0 \]
   \[ n = 5 \text{ or } n = 3 \]

3) \( 2v^2 + 5v - 25 = 0 \)
   \[ 2v^2 + 10v - 5v - 25 = 0 \]
   \[ 2v(v+5) - 5(v+5) = 0 \]
   \[ (v+5)(2v-5) = 0 \]
   \[ v+5 = 0 \text{ or } 2v-5 = 0 \]
   \[ v = -5 \text{ or } v = 5/2 \]

4) \( 2v^2 + v - 6 = 0 \)
   \[ 2v^2 + 4v - 3v - 6 = 0 \]
   \[ 2v(v+2) - 3(v+2) = 0 \]
   \[ (v+2)(2v-3) = 0 \]
   \[ v = -2 \text{ or } 3/2 \]

5) \( 3p^2 + 16p = 12 \)
   \[ 3p^2 + 16p - 12 = 0 \]
   \[ 3p^2 + 18p - 2p - 12 = 0 \]
   \[ 3p(p+6) - 2(p+6) = 0 \]
   \[ (p+6)(3p-2) = 0 \]
   \[ p = -6 \text{ or } 2/3 \]

6) \( 5n^2 + 4 = 21n \)
   \[ 5n^2 - 21n + 4 = 0 \]
   \[ 5n^2 - 20n - n + 4 = 0 \]
   \[ 5n(n-4) - 1(n-4) = 0 \]
   \[ (n-4)(5n-1) = 0 \]
   \[ n = 4 \text{ or } 1/5 \]
Solve by completing the square

Solve by completing the square.

\[ x^2 - 4x - 6 = 0 \]

Step 1 \[ x^2 + (-4x) = 6 \]
Step 2 \[ \left( \frac{-4}{2} \right)^2 = \]
Step 3 \[ x^2 - 4x + \frac{4}{2} = 6 \]
Step 4 \[ = \]
Step 5 \[ = \]
Step 6 \[ x - 2 = \sqrt{10} \text{ or } x - 2 = -\sqrt{10} \]
\[ x = \frac{\sqrt{10}}{2} \text{ or } x = \frac{-\sqrt{10}}{2} \]

Write in the form \[ x^2 + bx = c. \]
Find \( \left( \frac{b}{2} \right)^2 \).
Complete the square.
Factor and simplify.
Take the square root of both sides.
Write and solve two equations.

7) \[ k^2 + 8k - 9 = 0 \]
\[ \frac{8}{2} = 4^2 = 16 \]
\[ (k^2 + 8k + 16) - 16 - 9 = 0 \]
\[ (k + 4)^2 - 25 = 0 \]
\[ (k + 4)^2 = 25 \]
\[ k + 4 = \pm 5 \]
\[ k = 1, -9 \]

8) \[ n^2 + 2n - 21 = 0 \]
\[ (n^2 + 2n + 1) - 1 - 21 = 0 \]
\[ (n + 1)^2 = 22 \]
\[ n + 1 = \pm \sqrt{22} \]
\[ n = -1 \pm \sqrt{22} \]

9) \[ x^2 - 2x - 17 = -2 \]
\[ (x^2 - 2x + 1) - 1 - 17 = -2 \]
\[ (x - 1)^2 - 18 = -2 \]
\[ (x - 1)^2 = 16 \]
\[ x - 1 = \pm 4 \]
\[ x = 5, -3 \]

10) \[ n^2 + 2n - 12 = -3 \]
\[ (n^2 + 2n + 1) - 1 - 12 = -3 \]
\[ (n + 1)^2 - 13 = -3 \]
\[ (n + 1)^2 = 10 \]
\[ n + 1 = \pm \sqrt{10} \]
\[ n = -1 \pm \sqrt{10} \]
11) \(2x^2 + 16x + 1 = 3\)
\[
2(x^2 + 8x) + 1 = 3 \\
2(x^2 + 8x + 16) - 16(2) + 1 = 3 \\
2(x + 4)^2 - 31 = 3 \\
2(x + 4)^2 = 34 \\
(x + 4)^2 = 17 \\
x + 4 = \pm \sqrt{17} \\
x = -4 \pm \sqrt{17}
\]

12) \(3x^2 - 12x + 1 = 16\)
\[
3(x^2 - 4x) + 1 = 16 \\
3(x^2 - 4x + 4) - 4(2) + 1 = 16 \\
3(x - 2)^2 - 11 = 16 \\
3(x - 2)^2 = 27 \\
(x - 2)^2 = 9 \\
x - 2 = \pm 3 \\
x = 5, -1
\]

14) \(2x^2 + 12x - 46 = 0\)
\[
2(x^2 + 6x) - 46 = 0 \\
2(x^2 + 6x + 9) - 9(2) - 46 = 0 \\
2(x + 3)^2 - 64 = 0 \\
2(x + 3)^2 = 64 \\
(x + 3)^2 = 32 \\
x + 3 = \pm 4\sqrt{2} \\
x = -3 \pm 4\sqrt{2}
\]

15) \(6x^2 - 24x - 3 = 1\)
\[
6(x^2 - 4x) - 3 = 1 \\
6(x^2 - 4x + 4) - 4(2) - 3 = 1 \\
6(x - 2)^2 - 27 = 1 \\
6(x - 2)^2 = 28 \\
(x - 2)^2 = \frac{14}{3} \\
x - 2 = \pm \frac{\sqrt{42}}{3} \\
x = 2 \pm \frac{\sqrt{42}}{3}
\]

Solve with quadratic formula

Quadratic Formula:
\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

Use when it can't factor!
Word Problems

19. The length of a rectangular plot of land is 10 yards more than its width. If the area of the land is 600 square yards, find the dimensions of the plot of land.
20. A rectangular park has dimensions of $4x$ and $x+1$. There is a side walk around the perimeter of the park and inside the side walk is grass. The grass has dimensions of $x$ and $2x+1$. If the area of the side walk is $44 \text{ m}^2$, find the dimensions of the entire park.

\[
4x(x+1) - x(2x+1) = 44
\]

\[
4x^2 + 4x - 2x^2 - x = 44
\]

\[
2x^2 + 3x = 44
\]

\[
2x^2 + 3x - 44 = 0
\]

\[
x(2x + 11) - 4(2x + 11) = 0
\]

\[
(2x + 11)(x - 4) = 0
\]

\[
x = \frac{-11 \pm \sqrt{121 - 4(44)}}{4} = \frac{-11 \pm \sqrt{-87}}{4}
\]

21. Jon is hitting baseballs. When he tosses the ball into the air, his hand is 5 feet above the ground. He hits the ball when it falls back to a height of 4 feet. The height of the ball is given by $h(t) = 5 + 25t - 16t^2$, where $t$ is in seconds. How much time will pass before Jon hits the ball?

\[
-16t^2 + 25t + 5 = 4
\]

\[
-16t^2 + 25t + 1 = 0
\]

\[
16t^2 - 25t - 1 = 0
\]

\[
t = \frac{25 \pm \sqrt{(-25)^2 - 4(16)(-1)}}{2(16)}
\]

\[
t = \frac{25 \pm \sqrt{625}}{32}
\]

22. The height in feet of a bottle rocket is given by $h(t) = 160t - 16t^2$ where $t$ is the time in seconds. How long will it take for the rocket to return to the ground? What is the height after 2 seconds?

\[
-16t^2 + 160t = 0
\]

\[
-16t(t - 10) = 0
\]

\[
t = 0, 10
\]

\[
h(2) = 160(2) - 16(2)^2
\]

\[
h(2) = 256 \text{ ft}
\]

10 seconds to hit the ground.